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THE DISTRIBUTION OF THE CHARACTERISTICS OF THE MAXIMUM EXPECTED UTILITY PORTFOLIO BASED ON VAR: THE IMPACT OF INVESTOR'S RISK AVERSION COEFFICIENT

1. Introduction

Nowadays the problem of financial risk minimization is very important for financial institutions activity. Well known principle implies that the risk of portfolio which consists of two or more assets is not greater than the sum of risks of its components as the assets included in portfolio compensate the risk of each other. The first attempt to describe the methods of portfolio construction is made by Markowitz in 1952 [1]. This procedure is based on two characteristics of portfolio return and risk. The portfolio return is determined as expected return, namely the expectation of portfolio return, while the variance of the portfolio is taken as a risk measure. Markowitz gets the optimal portfolio weights by minimizing the portfolio risk for a given level of portfolio return or by maximizing expected return for a given level of portfolio risk.

Merton [2] shows that portfolios constructed in this way for different levels of return or risk form so-called efficient frontier, which is a parabola in the mean-variance space and hyperbola in the mean-standard deviation space. The main property of the efficient frontier is impossibility to increase the expected return without increasing the risk (variance) or to decrease risk (variance) without decreasing the expected return.

Generalization of Markowitz's theory is the maximization of expected quadratic utility function (see e.g. [3]). The main disadvantage of this method is its dependence from investor's risk aversion coefficient. If this coefficient approaches to $+\infty$ the investor is fully risk averse and the optimal choice for such investor is portfolio with minimal risk. Changing values of risk aversion coefficient from $+\infty$ to 0 we get the efficient frontier of portfolios. It is important to point out that there is no optimal method of rational choice of this coefficient.

The mentioned above methods of portfolio optimization are based on portfolio variance as a risk measure, which are heavily criticized recently because the variance is low informative and is two sided embranchment of risk. However, the indicator of portfolio risk should take positive values of loss function and give more information concerning portfolio risk than variance. A better candidate for this purpose is quantile based risk measures, which satisfy described above properties. Namely, these measures calculate portfolio risk on the basis of certain quantile of loss function distribution and fully describe the behaviour of random variable.

The most popular quantile based measure of risk is Value-at-Risk (henceforth VaR). This measure is recommended for risk calculation in banking by programs like Basel II, RiskMetrics, CAD II. In order to compute the VaR of portfolio we should first choose the confidence level α. Basel committee on banking supervision [4] recommends 99% for the value of the confidence level , whereas the values of {0.9, 0.95, 0.99, 0.999} also are used. In general case, the VaR at the confidence level α is equal to α-quantile of the loss function distribution that is the VaR at the confidence level α is the minimal level of losses with the probability 1-α. Due to simplicity in calculation and understandable results VaR is nowadays widely used on practice and a lot of works test the VaR properties (see e.g. [5]-[8]).

The problem of portfolio optimization on the basis of the VaR was first solved analytically by Alexander and Baptista [9]. In their paper authors find the solution of the problem of the portfolio VaR minimization without any conditions on portfolio return. It should be noted that minimum VaR portfolio also lies on the efficient frontier but has higher return comparing to the minimum variance portfolio for all values of $\alpha < 1$[1].

To generalize the results obtained in [9]-[10] we suggest considering the expected utility function based on VaR and the portfolio which maximizes the utility function. The main problem of expected utility function is its dependence on coefficient of risk aversion. In spite of the fact that to the problem of rational choice of coefficient of risk aversion paid much attention in financial literature ([11]-[13]) there is no unanimity around the method of doing this. This study aims to give some recommendation about choice of this coefficient based on properties of portfolio characteristics.

The rest of the paper is organized as follow. In the next section we present the basic notation and assumptions of the paper and solve the problem of maximization of expected utility based on VaR. Section 3 examines the problem of correctness of the sample estimator and discusses the estimators of the maximum expected utility portfolio along with their characteristics. Section 4 derives exact densities of portfolio characteristics estimators. Finally, using the properties of portfolio characteristics and monthly data from PFTS Ukrainian stock exchange market we give some recommendations concerning rational choice of investor's risk aversion coefficient in section 5.

2. Maximum expected utility portfolio and its characteristics

The main characteristic of a risky asset is its price at time $t(P_t)$. However, investor is more interested in price dynamic than in price itself. Financial literature employs the return as an indicator of price dynamic. We denote the return of risky asset at time point *t* as

$$X_t = 100 \ln \frac{P_t}{P_{t-1}}.$$

Let assume that we construct portfolio from k risky assets. Note that the number k is predetermined and we are not allowed to change its value. Such assumption is natural because we first should choose assets to invest our money before constructing portfolio. The $X_t = (X_{1t}, X_{2t}, ..., X_{kt})$ denotes k-dimensional vector of asset returns at time point t. Under the term "portfolio" we mean the vector $w = (w_1, Xw_2, ..., w_k)$, which consists of fractions of investors' wealth invested in corresponding assets such that $i'w = 1$, where i stands for the k-dimensional vector of ones. The w_i is the fraction of investors' wealth invested in i-th asset. The elements of vector w are called "portfolio weights".

In order to calculate the characteristics of the portfolio we need some assumptions on the asset returns behaviour. Financial literature frequently uses the assumption about normality of X_t, which is heavily criticized, since this assumption neglects some important properties of asset returns like asymmetry and heavy tailed distribution. Apparently, an application of these properties would make the outcomes more realistic but we are forced to use numerical instead of analytical methods of research.

To test exact statistical properties of portfolio we first need to solve the problem of portfolio construction analytically. The latter implies that the assumption of normality is adequate in the context of this paper. Note that monthly and yearly returns have distributions very close to normal [14]. Additionally, the impact of heavy tails on the portfolio properties can be weakened by additional diversification [5].

We also assume that X_t has k-dimensional normal distribution with the mean vector μ and the covariance matrix $\sum$. Let Xwt be the portfolio return at time point t. Then the expected return of portfolio is $Rw = E(Xwt) = \mu' w$ the variance – $Vw = Var(Xwt) = w' \sum w$ and the Var at confidence level α (from assumption of normality of Xt) $-Mw = z\alpha\sqrt{Var(X_{wt})}$ - $E(Xwt) = z_\alpha \sqrt{w' \sum w - \mu' w}$, where $z_\alpha = -\Phi^{-1}(1-\alpha)$ – α-quantile of the standard normal distribution.

In contrast to the classical definition of the expected quadratic utility function, we use the utility function based on VaR

$$U(w) = R_w - \frac{\beta}{2} M_w = \mu' w - \frac{\beta}{2}\left(z_\alpha \sqrt{w'\Sigma w} - \mu' w\right), \tag{1}$$

where β is the investor's risk aversion coefficient. The utility function (1) fits better the recommendations of Basel committee than classical definition[1]. Moreover, the interpretation of the utility (1) is better than for classical definition because the interpretation of the difference between the expected return of portfolio and its VaR is more reasonable than the difference between expected return of portfolio and its variance.

We consider the optimization problem

$$U(w) \to \max, \text{ subject to } i' w = 1. \tag{2}$$

We allow the possibility of short sales by absence of the condition of positive portfolio weights ($w_i \geq 0$). Including this condition in (2) makes the use of analytical methods of solution impossible. The values of β should be strictly positive because in case of $\beta = 0$ investor maximizes the return without paying attention to risk. Such problem cannot be solved without additional conditions on w. The utility also can be negative. If investor is interested in the positive utility the range for β is $\left(0;\ 2\frac{\mu' w}{z_\alpha \sqrt{w'\Sigma w} - \mu' w}\right]$.

Proposition 1. Let construct portfolio with k assets. X_t denotes k-dimensional vector of asset returns at time point t. We assume that $X_t \sim Nk(\mu, \Sigma)$. Then the solution of optimization problem exist if $\tilde{\beta}^2 z_\alpha^2 > s$, where $\tilde{\beta} = \beta / (\beta + 2)$, $s = \mu' R \mu$, $R = \Sigma^{-1} - \frac{\Sigma^{-1} i i' \Sigma^{-1}}{i' \Sigma^{-1} i}$ and has the form

$$w_{UVaR} = w_{GMV} + \frac{\sqrt{V_{GMV}}}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - s}} R\mu, \tag{3}$$

where $w_{GMV} = \frac{\Sigma^{-1} i}{i' \Sigma^{-1} i}$ – weight vector of global minimum variance portfolio, $V_{GMV} = \frac{1}{i' \Sigma^{-1} i}$ – the variance of portfolio w_{GMV}.

Proof. Using simple algebraic transformation the optimization problem (2) can be rewritten in the following way

$$U(w) = \left(1 + \frac{\beta}{2}\right)\left(\mu' w - \frac{\beta}{2+\beta} z_\alpha \sqrt{w'\Sigma w}\right) \to \max, \text{ subject to } i' w = 1,$$

which is equivalent (because $\beta \geq 0$) to the problem

$$\tilde{\beta}\, z_\alpha \sqrt{w'\Sigma w} - \mu' w \to \min, \text{ subject to } i' w = 1. \tag{4}$$

Denoting by $\tilde{z}_\alpha = \tilde{\beta}\, z_\alpha$ we get that (4) is equivalent to the problem of VaR minimization of Alexander and Baptista [9], which implies the necessary proposition.

[1] Alexander and Baptista [9] substantiate the possibility of such utility function.

Note that if β tends to infinity, the portfolio weights (3) approach to portfolio weights with the minimum VaR (see e.g. [10]). As a consequence we can calculate the characteristics of portfolio $\mathbf{w}_{UVaR}$ (expected return, variance, VaR).

Corollary 1. Under conditions of proposition 1 the characteristics of portfolio $\mathbf{w}_{UVaR}$ have the form

$$R_{UVaR} = w'_{UVaR}\,\mu = R_{GMV} + \frac{\sqrt{V_{GMV}}}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - s}}\, s\,, \tag{5}$$

$$V_{UVaR} = w'_{UVaR}\,\Sigma w_{UVaR} = \frac{\tilde{\beta}^2 z_\alpha^2}{\tilde{\beta}^2 z_\alpha^2 - s} V_{GMV}\,, \tag{6}$$

$$M_{UVaR} = \frac{\tilde{\beta}\, z_\alpha^2 - s}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - s}}\sqrt{V_{GMV}} - R_{GMV}\,, \tag{7}$$

where $R_{GMV} = \dfrac{\mu'\Sigma^{-1} i}{i'\Sigma^{-1} i}$ – expected return of global minimum variance portfolio w_{GMV}.

From (3), (5)-(7) can be seen that weights and characteristics of portfolio w_{UVaR} depend on parameters of asset returns distribution. Substitution of these parameters into corresponding formulae gives full information for investor about constructed portfolio. On practice the parameters μ and Σ are unknown. That's why the investor has no possibility to use the previous results straightforward. Thus, he should somehow estimate these parameters.

3. The adequacy of sample portfolio weights estimator

Our analysis demonstrates that the results of proposition 1 cannot be used straightforward and the unknown parameters of asset returns distribution μ and Σ should be estimated. For these purposes we make use of the sample estimators. Let an independent random sample of the asset returns $X_1, X_2, ..., X_n$ is available. The sample estimators of the mean vector μ and the covariance matrix Σ are expressed as

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i\,,\ \hat{\Sigma} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})(X_i - \hat{\mu})'\,. \tag{8}$$

Replacing the unknown parameters μ and Σ by their sample estimators (8) in (3), (5)-(7), we get the sample estimators of the weights and the characteristics of portfolio $\mathbf{w}_{UVaR}$

$$\hat{w}_{UVaR} = \hat{w}_{GMV} + \frac{\sqrt{\hat{V}_{GMV}}}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - \hat{s}}}\hat{R}\hat{\mu}\,, \tag{9}$$

$$\hat{R}_{UVaR} = \hat{R}_{GMV} + \frac{\sqrt{\hat{V}_{GMV}}}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - \hat{s}}}\hat{s}\,, \tag{10}$$

$$\hat{V}_{UVaR} = \frac{\tilde{\beta}^2 z_\alpha^2}{\tilde{\beta}^2 z_\alpha^2 - \hat{s}}\hat{V}_{GMV}\,, \tag{11}$$

$$\hat{M}_{UVaR} = \frac{\tilde{\beta}\, z_\alpha^2 - s}{\sqrt{\tilde{\beta}^2 z_\alpha^2 - s}}\sqrt{\hat{V}_{GMV}} - \hat{R}_{GMV}\,, \tag{12}$$

where

$$\hat{R} = \hat{\Sigma}^{-1} - \frac{\hat{\Sigma}^{-1} i i'\hat{\Sigma}^{-1}}{i'\hat{\Sigma}^{-1} i}\,,\ s = \hat{\mu}'\hat{R}\hat{\mu}\,,\ \hat{w}_{GMV} = \frac{\hat{\Sigma}^{-1} i}{i'\hat{\Sigma}^{-1} i}\,,\ \hat{R}_{GMV} = \frac{\hat{\mu}'\hat{\Sigma}^{-1} i}{i'\hat{\Sigma}^{-1} i}\,,\ \hat{V}_{GMV} = \frac{1}{i'\hat{\Sigma}^{-1} i}\,.$$

From proposition 1 the necessary and sufficient condition for existing the portfolio with the maximum utility is $\tilde{\beta}^2\ z_\alpha^2 > s$. The implementation of this condition does not ensure the fulfillment of the condition $\tilde{\beta}^2\ z_\alpha^2 > \hat{s}$ which is necessary for sample estimators (9)-(12) to be adequate. This fact is implied by the randomness of $\hat{s}$. The distribution of $\hat{s}$ is found by Bodnar and Schmid [15] under assumption of normality of X_t. In [15] it is shown that the random variable $\frac{n(n-k+1)}{(n-1)(k-1)}\hat{s}$ has non-central Fisher distribution with $k-1$ and $n-k+1$ degrees of freedom, and non-centrality parameter ns. The problem of adequacy of sample estimator of portfolio weights with minimum VaR is studied in [10] and, as it is shown, the probability of sample estimator adequacy is close to 1 for values of α higher than *0.9*. In our study we need to investigate the probability of fulfillment of the condition $\tilde{\beta}^2\ z_\alpha^2 > \hat{s}$ depending on β and we can use the same algorithm as in [10].

We calculate upper and lower bound for the probability $P\left\{\tilde{\beta}^2\ z_\alpha^2 > \hat{s}\right\}$ using monthly data for four Ukrainian companies (*k*=4) from PFTS Ukrainian stock exchange market: CEEN, ALMK, UTLM and MSICH, for the time period from 04.2009 to 10.2012 (*n*=42). Our calculation gives us $\hat{s} = 0.15\ 7845$. We also take α *=0.95* and changing the value of β from *0* to +∞. The bounds received are depicted on Figure 1. Obtained results allow us to conclude that the probability that the sample estimator of portfolio weights is adequate is close to one and increase with the value of β for values $\beta \geq 1$. The lower bound of the considered probability is *0.938* for $\beta = 1$ and is larger than *0.999* for $\beta = 4$. Thus, the probability of adequacy of sample estimator of portfolio weights $\mathbf{w}_{UVaR}$ is high for values of $\beta \geq 1$ which means that we can use such estimator not only for theoretical purposes but also in practice.

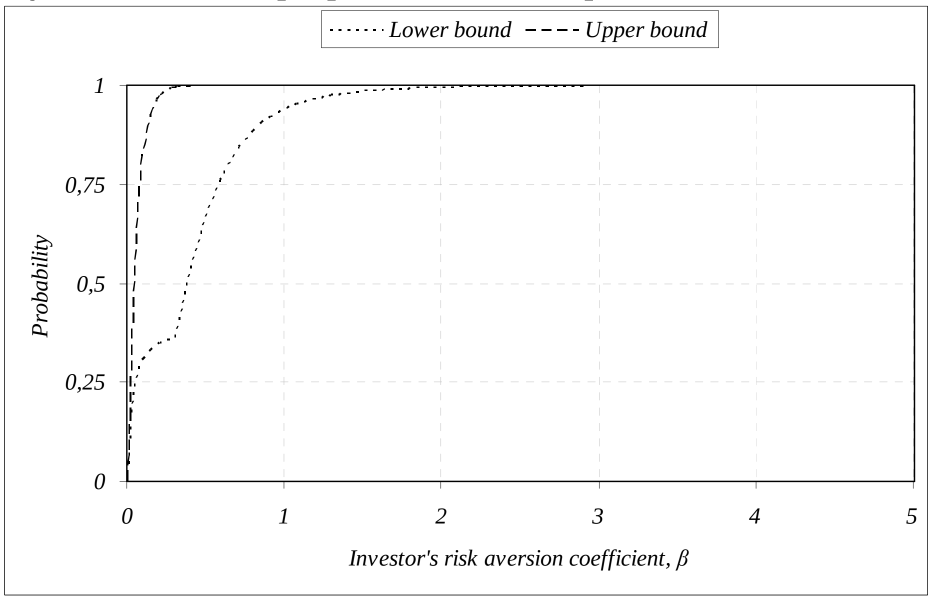


Figure 1. The bounds of 95% confidence interval for probability $P(\hat{s} < \tilde{\beta}^2 z_\alpha^2)$ *(α=0.95).*

4. The distribution of portfolio characteristics sample estimators

In this section we examine the properties of portfolio characteristics w_{UVaR} sample estimators. Since the sample estimators (9) are random variables then also the estimators of portfolio characteristics (10)-(12) are random variables. The best way to describe the properties of random variable is to use their distribution. That's why we need to find the exact distribution of random variables (10)-(12). Note that the unconditional distributions are not appropriate in our case because the estimators (10)-(12) are tractable only if $\tilde{\beta}^2\ z_\alpha^2 > \hat{s}$. We consider the distributional properties of portfolio characteristics estimators under the condition that $\tilde{\beta}^2\ z_\alpha^2 > \hat{s}$. Let

$$a(s^*)=\frac{s^*}{\sqrt{\tilde\beta^2 z_\alpha^2 - s^*}}\sqrt{\frac{V_{GMV}}{n-1}},\ b(s^*)=\frac{\tilde\beta^2 z_\alpha^2}{\tilde\beta^2 z_\alpha^2 - s^*}\frac{V_{GMV}}{n-1},$$

$$c(s^*)=\frac{\tilde\beta\, z_\alpha^2 - s^*}{\sqrt{\tilde\beta^2 z_\alpha^2 - s^*}}\sqrt{\frac{V_{GMV}}{n-1}},\ \tilde s^*=\frac{1+n/(n-1)s^*}{n}V_{GMV},$$

$$M(x;m,a,b_1,b_2)=\frac{1}{\sqrt{2\pi}|b_1|}\int_0^\infty t^{m-1}\exp\left\{-\frac{1}{2}\left(\frac{1}{b_1^2}+\frac{1}{b_2^2}\right)\left(t-(x-a)\frac{b_2^2}{b_2^2+b_1^2}\right)^2\right\}dt.$$

Theorem 1. Let $X_1, X_2, ..., X_n$ be independent random vectors and $X_i \sim N(\mu,\Sigma)$ X_i~N(μ, Σ) for all i in $1,...,n$, Σ –is positive definite, k – the number of assets in portfolio and $n>k$. Then

a) $$f_{\hat R_{UVaR}|\hat s<\tilde\beta\, z_\alpha^2}=K(\tilde\beta\, z_\alpha^2)\int_0^{\tilde\beta\, z_\alpha^2} f_{k-1,n-k+1;ns}\left(\frac{n(n-k+1)}{(n-1)(k-1)}s^*\right)f_{\hat R^*_{UVaR}}(x\,|\,s^*)ds^*,$$

b) $$f_{\hat V_{UVaR}|\hat s<\tilde\beta\, z_\alpha^2}=K(\tilde\beta\, z_\alpha^2)\int_0^{\tilde\beta\, z_\alpha^2} f_{k-1,n-k+1;ns}\left(\frac{n(n-k+1)}{(n-1)(k-1)}s^*\right)f_{\hat V^*_{UVaR}}(x\,|\,s^*)ds^*,$$

c) $$f_{\hat M_{UVaR}|\hat s<\tilde\beta\, z_\alpha^2}=K(\tilde\beta\, z_\alpha^2)\int_0^{\tilde\beta\, z_\alpha^2} f_{k-1,n-k+1;ns}\left(\frac{n(n-k+1)}{(n-1)(k-1)}s^*\right)f_{\hat M^*_{UVaR}}(x\,|\,s^*)ds^*,$$

where $K(x)=\frac{n(n-k+1)}{(n-1)(k-1)}\frac{1}{F_{k-1,n-k+1;ns}\left(\frac{n(n-k+1)}{(n-1)(k-1)}x\right)}$, $F_{d_1,d_2;\lambda}(x)$ and $f_{d_1,d_2;\lambda}(x)$ distribution function and density of non-central Fisher distribution with d_1 and d_2 degrees of freedom and non-centrality parameter λ correspondingly,

$$f_{\hat R^*_{UVaR}}(x)=\frac{1}{2^{\frac{n-k-2}{2}}a(s^*)^{n-k}\Gamma((n-k)/2)}\exp\left\{-\frac{(x-R_{GMV})^2}{2(a(s^*)^2+\tilde s^*)}\right\}M(x;n-k,R_{GMV},\sqrt{\tilde s^*},a(s^*)),$$

$$f_{\hat V^*_{UVaR}}(x)=\frac{1}{(2b(s^*))^{\frac{n-k}{2}}\Gamma((n-k)/2)}x^{\frac{n-k-2}{2}}\exp\left\{-\frac{x}{2b(s^*)}\right\},$$

$$f_{\hat M^*_{UVaR}}(x)=\frac{1}{2^{\frac{n-k-2}{2}}c(s^*)^{n-k}\Gamma((n-k)/2)}\exp\left\{-\frac{(x+R_{GMV})^2}{2(c(s^*)^2+\tilde s^*)}\right\}M(x;n-k,-R_{GMV},-\sqrt{\tilde s^*},c(s^*))$$

the density functions of appropriate portfolio characteristics estimators under condition that $\hat s = s^*$.

Proof. The proof of the theorem is equivalent to the proof of theorem 3 in [10].

As a consequence of theorem 1 we can get the results of theorem 3 in [3] substituting instead of $\tilde\beta$ one or equivalent instead of $\beta \pm \infty$.

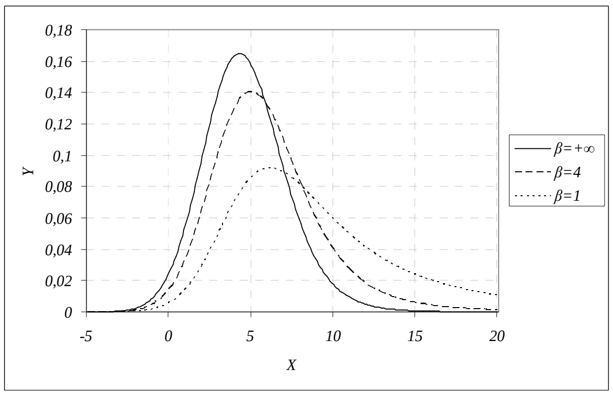
We use the outcomes of theorem 1 in order to investigate the impact of investor's risk aversion coefficient β on the density of portfolio characteristics estimators. We use the sample counterparts from the monthly data for four Ukrainian companies from PFTS Ukrainian stock exchange market ($k=4$) for the unknown parameters: CEEN, ALMK, UTLM and MSICH, for the time period from 04.2009 to 10.2012 ($n=42$). We get

$$\mu = (1.598,\ 0.029,\ 0.324,\ 4.575)',\ \Sigma = \begin{pmatrix} 276.87 & 244.64 & 129.25 & 184.42 \\ 244.64 & 440.17 & 177.33 & 231.48 \\ 129.25 & 177.33 & 198.78 & 111.72 \\ 184.42 & 231.48 & 111.72 & 226.27 \end{pmatrix}.$$

The high variance of Ukrainian stock market is a consequence of low liquidity of the market. Substituting these estimators into formulae for R_{GVM}, V_{GVM} and s, we get

$$R_{GVM} = 2.70094,\ V_{GVM} = 150.402,\ s = 0.15785.$$

In Figure 2 the density of $\hat{R}_{UVaR}$ (left) and $\hat{M}_{UVaR}$ (right) under condition that $\tilde{\beta}^2 z_\alpha^2 > \hat{s}$ are plotted. We use the value 0.95 for confidence level α and $\beta \in \{1, 4, +\infty\}$.



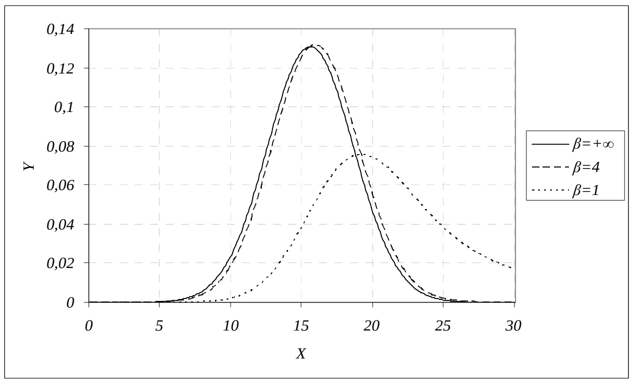


Figure 2. The densities of $\hat{R}_{UVaR}$ (left) and $\hat{M}_{UVaR}$ (right) under condition that $\tilde{\beta}^2 z_\alpha^2 > \hat{s}$ with $\alpha = 0.95$ and $\beta = \{1, 4, +\infty\}$.

The expected return of portfolio $\mathbf{w}_{UVaR}$ is more sensitive to the values of β than the *VaR* for $\beta \in [4, +\infty)$ (see figure 2). Changing β in this interval the portfolio expected return increases and its density changes from almost symmetric to the positive skewed. The right tail also becomes heavy. Contrary to the portfolio expected return, the risk of the portfolio is almost the same along the interval. Another situation is obtained in the interval $\beta \in [1, 4]$. The change in density of portfolio expected return is mostly proportional to the previous case but the changes in density of portfolio risk are crucial. The distribution of portfolio risk sharply becomes positive skewed and the right tail of the distribution becomes very heavy. Taking into account mentioned above we can formulate some recommendation about rational choice of risk aversion coefficient. First, the use of high coefficients of risk aversion is inappropriate in Ukrainian market. The distribution of portfolio risk is almost the same for medium values of β and the distribution of portfolio expected return moves to the right. Second, the low values of β also should be avoided because the distribution of portfolio risk becomes positively skewed and gets very heavy right tail. Thus, one should perform the appropriate analysis of distributions of hypothetical portfolio characteristics to choose the coefficient β which satisfies the investor's expectation concerning the distributional properties of portfolio characteristics.

5. Conclusions

The paper examines properties of portfolio characteristics with maximum expected utility based on Value-at-Risk. The use of this risk measure in portfolio theory is fully consistent with recommendations of the main banking documents. From the theoretical point of view application of expected utility function for portfolio constructing is a generalization of the portfolio constructing problem with minimum risk and given level of portfolio return. The main drawback of this method is impossibility of optimal choice of coefficient β, which describes investor's attitude to risk. Note that this coefficient also has an impact on the adequacy of sample estimator of portfolio weights. It is shown that for values of β which are close to zero, the adequacy of the estimator is very questionable. But for the values higher than one the probability of adequacy of this estimator is close to one. For example, for $\beta = 1$ the lower bound of probability is equal to *0.938*, and for $\beta = 4$ – larger than *0.999*. Therefore we can use such an estimator not only for theoretical purposes but also on practice.

For the investigation of impact of investor's risk aversion coefficient on the main portfolio characteristics the exact distribution of these characteristics are found. Changing the coefficient β in the range $[1, +\infty)$ the graphs of the densities of portfolio characteristics are considered. Based on these graphs some recommendations concerning the rational choice of β are given. For $\beta \in [4, +\infty)$ the expected return of portfolio is more sensible to the changes in β than the risk since the density of portfolio risk is almost the same for all $\beta \in [4, +\infty)$. It indicates that the use of high values for β is inappropriate because the increase of β leads to substantial decrease of portfolio expected return while the portfolio risk remains almost unchanged. For $\beta \in [1, 4)$ the portfolio expected return changes in the same way as in the previous case. However the changes in portfolio risk distribution are crucial. It becomes strong positive skewed and gets very heavy right tail. Hence the use of low values for β is also inappropriate due to the sharp increase of portfolio risk together with moderate increase of portfolio expected return.

It should be noted that the results of the present paper can be extended to the case of non-normal distribution. In this case the Monte Carlo simulation study should be used in order to plot the corresponding distributions.

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