УДК 330.46

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# MODELING MULTIVARIATE NONSTATIONARY TIME SERIES OF ECONOMIC DYNAMICS BASED ON FOKKER-PLANCK EQUATION

The subject of economic dynamics is a simulation of behavior of economic systems under the influence of internal and external factors in order to analyze balance, control and prediction of the evolution of economic systems. Mathematical models of economic dynamics are the formal reflection of the many economic scenarios development. This is usually determined models which are based on some economic concepts. Under the economic dynamics also understand the dynamic series - series of numbers that characterize the change of the social or economic event that is economic time series (ETS) [1]. Really researched ETS - nonstationary and almost always multi sign nature, their reliable reflection in the economic - mathematical models is possible only on condition taking into account the complex inherent features of the most significant characteristics. Currently, the development of mathematical methods for the analysis of nonstationary multivariate processes has great practical importance and demand in the economy. This is due to the need to improve the accuracy of predictions and, in particular, the price indicators of financial and commodity markets.

Modern economics has in its arsenal a large number of methods for analyzing ETS and predicting socio - economic indicators: statistical [2], determined [3], econophysics methods, phase analysis, wavelet - analysis, spectral analysis [4-7 ], adaptive forecasting methods - neural networks, genetic algorithms, group method of data handling, method " caterpillar" [8,9]. Most of these approaches are used only for one-dimensional and stationary ETS. Constructing a statistically correct economic - mathematical models of multivariate nonstationary ETS and methods of analysis is an actual problem. The theoretical basis of these models is a multi-dimensional Fokker - Planck equation [10,11]. The purpose of this study is to develop mathematical models of nonstationary multi ETS for their analysis, forecasting and decision-making in financial and commodity markets on the basis of the author's concept of multivariate modeling of dynamic systems [12].

1. The relationship between the continuity equation and the Fokker - Planck equation. Dynamic System (DS) is characterized by state variables $\mathbf{x}=(x_1,x_2,...,x_m)$ – system of economic performance and function of the state $\phi(\mathbf{x})=\phi(x_1,x_2,...,x_m)$ – probability density function (PDF), considered in the phase space $\Gamma$ points $\mathbf{x}$. Measure $d\Gamma_m=\phi(x_1,x_2,...,x_m)\prod_{i=1}^{m}dx_i$ is the number of states ETS of specified intervals $dx_i$.

State variables are standardize:

$$x_i = \frac{z_i - z_{\min}}{z_{\max} - z_{\min}}, \quad x_i \in [0,1],$$

where $z_i$, $z_{\min}$, $z_{\max}$ – observed, minimal, maximal value levels of economic series.

Given that the ETS is nonstationary, specified function status should be considered taking into account depending on the time $\phi(\mathbf{x},t)$. Let under the influence of socio-economic factors, state variables vary with speed

$$V(\mathbf{x},t) = \left\{ V_{x_1}(x_1, x_2, ..., x_m; t), V_{x_2}(x_1, x_2, ..., x_m; t), ..., V_{x_m}(x_1, x_2, ..., x_m; t) \right\}.$$

Between functions $\phi(\mathbf{x},t)$ and $V(\mathbf{x},t)$ there is a relationship in the form of conservation law of matter in its differential form (continuity equation):

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = -div\left(\phi(\mathbf{x},t) \cdot V(\mathbf{x},t)\right). \tag{1}$$

Equation (1) in the economic interpretation is the equation of balance of financial and material flows. The meaning of this equation is that the change in the probability of states per unit phase space per unit time is equal to the total flux passing through an arbitrary closed surface in a neighborhood of $\mathbf{x}$ during this time.

The basis of the evolution of any complex economic system is the interaction between the two factors of different nature - the growth and dissipation. Growth model can be based on different economic concepts that reflect the mechanisms of the socio - economic factors: supply and demand, competition, positive feedback and more. The effect of these mechanisms leads to a local temporary and spatial heterogeneity, increase the intensity of flow redistribution between sectors. Dissipation, in contrast, is accompanied alignment concentrations decrease in the intensity of flows, increased uncertainty and entropy, which eventually leads to the degradation of the economic system. Naturally we assume that the value of the flow rate of dissipation at a point is inversely proportional to the gradient of the probabilities of states at this point. Then the evolution of the DS can be represented by the following equation:

$$\dot{x}_i = V_{x_i}(\mathbf{x},t) = u_i(\mathbf{x},t) - \frac{1}{2\phi(\mathbf{x},t)} \sum_{j=1}^{m} \frac{\partial}{\partial x_j}\left[ b_{ij}(\mathbf{x},t) \cdot \phi(\mathbf{x},t) \right], \tag{2}$$

where $u_i(\mathbf{x},t)$, $b_{ij}(\mathbf{x},t)$ – measurable function for each $t \in [0,T]$, a $T$ – observed period.

Substituting (2) into equation (1), we obtain the Fokker-Planck equation:

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} + \sum_{i=1}^{m} \frac{\partial}{\partial x_i}\left[ u_i(\mathbf{x},t)\phi(\mathbf{x},t) \right] - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2}{\partial x_i \partial x_j}\left[ b_{ij}(\mathbf{x},t)\phi(\mathbf{x},t) \right] = 0. \tag{3}$$

Fokker - Planck equation (3) is closely connected with the system of stochastic differential equations:

$$\dot{x}_i(t) = u_i(\mathbf{x},t) + \sum_{j=1}^{m} b_{ij}(\mathbf{x},t)\xi_j(t), \tag{4}$$

where $u_i$, $b_{ij}$ – deterministic functions; $\xi_i(t)$ – independent random processes such as white noise, that is $\mathrm{M}\xi_i(t) = 0, \mathrm{M}\xi_i(t)\xi_i(t+\tau) = \delta(\tau), \mathrm{M}\left(\xi_i(t) \cdot \xi_i(t)\right) = 0$ at $i \neq j$. Averaging equation (4) for the set $\dot{\boldsymbol{x}}$, we obtain:

$$u_i(\boldsymbol{x},t) = m(\dot{x}_i | x_1, x_2, ..., x_m; t) = \frac{1}{\varphi(\boldsymbol{x},t)} \int \dot{x}_i F(\boldsymbol{x}, \dot{\boldsymbol{x}}; t) d\dot{x}_1 d\dot{x}_2 ... d\dot{x}_m, \tag{5}$$

where $F(\boldsymbol{x}, \dot{\boldsymbol{x}}; t) = F(x_1, x_2, ..., x_m, \dot{x}_1, \dot{x}_2, ..., \dot{x}_m; t)$ – PDF dimensionality $2m$.

Last explains statistical content of functions $u_i(\boldsymbol{x},t)$. By definition, the expression (5) - is the conditional mathematical expectation of speed that is a function $u_i$ – component of the average local speed of the field. Equation (3) is supplemented by boundary and initial conditions $\varphi(0,t)=\varphi(1,t)=0$, $\varphi(\boldsymbol{x},0)=\varphi_0(\boldsymbol{x})$, together with the kinetic equations (2) completely determine the state of a stochastic dynamical system to each moment of time $t$.

*2. A mathematical model of the stock exchange*. Trading on the stock exchange are characterized by a number of indicators, including basic and most informative are: $x_1$ – average quotation price, $x_2$ – physical volume of sales, $x_3$ – spread.

Consider the three subsystems (information channels) of sales - "price-volume", "price-spread", "volume-spread", which generate two-dimensional random processes with PDF $\varphi_{12}(x_1,x_2;t)$, $\varphi_{13}(x_1,x_3;t)$, $\varphi_{23}(x_2,x_3;t)$ respectively. These PDF satisfy three two-dimensional Fokker-Planck equation:

$$\frac{\partial\varphi_{ij}}{\partial t}+\frac{\partial}{\partial x_i}(u_{ij}\varphi_{ij})+\frac{\partial}{\partial x_j}(v_{ij}\varphi_{ij})-\frac{\mu_{ij}}{2}\frac{\partial^2\varphi_{ij}}{\partial x_i^2}-\eta_{ij}\frac{\partial^2\varphi_{ij}}{\partial x_i\partial x_j}-\frac{\kappa_{ij}}{2}\frac{\partial^2\varphi_{ij}}{\partial x_j^2}=0 \quad , \tag{6}$$

where $u_{ij}=u_{ij}(x_i,x_j;t)$, $v_{ij}=v_{ij}(x_i,x_j;t)$ – drift functions; $\mu_{ij}$, $\eta_{ij}$, $\kappa_{ij}$ – elements of diffusion matrix of two-dimensional process.

We assume that the two-dimensional processes are not heteroscedastic, that is $\mu_{ij}=\mu_{ij}(t)$, $\eta_{ij}=\eta_{ij}(t)$, $\kappa_{ij}=\kappa_{ij}(t)$ and $u_{ij}$, $v_{ij}$ – this one-parameter non-linear function of the form:

$$\begin{cases} u_{ij}=\lambda_{ij}x_i(1-x_j) \\ v_{ij}=\chi_{ij}x_j(1-x_i), \end{cases} \tag{7}$$

where $\lambda_{ij}=\lambda_{ij}(t)$, $\chi_{ij}=\chi_{ij}(t)$ – model parameters.

Drift models (7) based on the assumption of the presence of competition and limited resources. The structure of the model equations (7) corresponds to the non-linear paradigm of financial markets and is consistent with the results of empirical studies, including the approach of making investment decisions based on technical analysis.

To identify the parameters of the model, consider the evolution of these characteristics ETS: mathematical expectations $m_{x_i}(t)$, dispersions $\sigma^2_{x_i}(t)$ and covariance $cov_{x_i,x_j}(t)$:

$$m_{x_i}(t)=\int x_i\psi_i(x_i,t)dx_i,\quad \sigma^2_{x_i}=\int(x_i-m_{x_i})^2\psi_i(x_i,t)dx_i,$$
$$R_{ij}(t)=\mathrm{cov}_{x_i,x_j}(t)=\int(x_i-m_{x_i})\cdot(x_j-m_{x_j})\psi_{ij}(x_i,x_j;t)dx_idx_j, \tag{8}$$

where $\psi_i(x_i,t)$ – one-dimensional (partial) PDF of two-dimensional processes.

Since the one-dimensional PDF received from $\varphi_{ij}(x_i,x_j;t)$ with equations:

$$\psi_i(x_i,t)=\int\varphi_{ij}(x_i,x_j;t)dx_j,\quad \psi_j(x_j,t)=\int\varphi_{ij}(x_i,x_j;t)dx_i\ ,$$

then integrating equation (6) sequentially by variable $x_i, x_j$ given the fact that within the region of integration PDF is zero, we obtain two-dimensional Fokker - Planck equation for each of the three two dimensional processes:

$$\frac{\partial\psi_i}{\partial t}+\frac{\partial}{\partial x_i}(U_{ij}\psi_i)-\frac{\mu_{ij}}{2}\frac{\partial^2\psi_i}{\partial x_i^2}=0,\quad \frac{\partial\psi_i}{\partial t}+\frac{\partial}{\partial x_j}(V_{ij}\psi_j)-\frac{\kappa_{ij}}{2}\frac{\partial^2\psi_j}{\partial x_j^2}=0, \tag{9}$$

where $U_{ij}=\dfrac{1}{\psi_i}\int u_{ij}(x_i,x_j;t)\varphi_{ij}(x_i,x_j;t)dx_j$, $V_{ij}=\dfrac{1}{\psi_i}\int v_{ij}(x_i,x_j;t)\varphi_{ij}(x_i,x_j;t)dx_i$ or based on accepted models (7) -

$$U_{ij}(x_i,t)=\lambda_{ij}x_i\left(1-e_{ij}(x_i,t)\right),\quad V_{ij}(x_j,t)=\chi_{ij}x_j\left(1-f_{ij}(x_j,t)\right), \tag{10}$$

where $e_{ij}(x_i,t)$, $f_{ij}(x_j,t)$ – conditional mathematical expectations, which are defined by the formulas:

$$e_{ij}(x_i,t)=\frac{1}{\psi_i}\int x_j\varphi_{ij}(x_i,x_j;t)dx_j,\quad f_{ij}(x_j,t)=\frac{1}{\psi_j}\int x_i\varphi_{ij}(x_i,x_j;t)dx_i. \tag{11}$$

The equation of evolution of mathematical expectation indicators ETS we will find considering their definitions (8) and consistency dimensional distributions of the Fokker - Planck equation (9):

$$\frac{dm_{x_i}}{dt}=\int x_i\frac{\partial\psi_i(x_i,t)}{\partial t}dx_i=\int x_i\left(\frac{\mu_{ij}}{2}\frac{\partial^2\psi_i}{\partial x_i^2}-\frac{\partial}{\partial x_i}\left(U_{ij}\psi_i\right)\right)dx_i,$$

$$\frac{dm_{x_j}}{dt}=\int x_j\frac{\partial\psi_j(x_j,t)}{\partial t}dx_j=\int x_j\left(\frac{\kappa_{ij}}{2}\frac{\partial^2\psi_j}{\partial x_j^2}-\frac{\partial}{\partial x_j}\left(V_{ij}\psi_j\right)\right)dx_j,$$

which after integration by parts, taking into account the zero boundary conditions for PDF and expressions (10) become equations:

$$\frac{dm_{x_i}}{dt}=\int U_{ij}(x_i,t)\psi_i(x_i,t)dx_i=\lambda_{ij}\left(m_{x_i}-m_{x_i}m_{x_j}-cov_{x_i,x_j}\right)=\langle U\rangle_t,$$

$$\frac{dm_{x_j}}{dt}=\int V_{ij}(x_j,t)\psi_j(x_j,t)dx_j=\chi_{ij}\left(m_{x_j}-m_{x_i}m_{x_j}-cov_{x_i,x_j}\right)=\langle V_{ij}\rangle_t. \tag{12}$$

Considering the derivative of the expectation as finite difference (growth rate ETS), and the expected value as the average levels of ETS, it is easy to make sure that $\dot m_{x_i}=m_{\dot x_i}$ $\dot m_{x_j}=m_{\dot x_j}$. Then from (12) that the rate of change of average values of ETS is the average velocity of the field. This is the statistical meaning of equations (12) for determining statistical parameter estimation $\lambda_{ij}$ and $\chi_{ij}$:

$$\lambda_{ij}=\frac{m_{\dot x_i}}{m_{x_i}(1-m_{x_j})-\mathrm{cov}_{x_i,x_j}},\quad \chi_{ij}=\frac{m_{\dot x_j}}{m_{x_j}(1-m_{x_i})-\mathrm{cov}_{x_i,x_j}}. \tag{13}$$

Under stochastic processes $\dot x_i(t)$ и $\dot x_j(t)$ we will understand ranks of the first finite difference of economic indicators $\dot x(t)=\Delta(t)=x(t+1)-x(t)$.

Similarly, the evolution equation of the variance can be written as:

$$\frac{d\sigma^2_{x_i}}{dt}=\int\left(x_i-m_{x_i}\right)^2\frac{\partial\psi_i}{\partial t}dx_i=\int\left(x_i-m_{x_i}\right)^2\left(\frac{\mu_{ij}}{2}\frac{\partial^2\psi_i}{\partial x_i^2}\quad\frac{\partial}{\partial x_i}\left(U_{ij}\psi_i\right)\right)dx_i,$$

$$\frac{d\sigma^2_{x_j}}{dt}=\int\left(x_j-m_{x_j}\right)^2\frac{\partial\psi_j}{\partial t}dx_j=\int\left(x_j-m_{x_j}\right)^2\left(\frac{\kappa_{ij}}{2}\frac{\partial^2\psi_j}{\partial x_j^2}\quad\frac{\partial}{\partial x_j}\left(V_{ij}\psi_j\right)\right)dx_j.$$

After integration we obtain:

$$\frac{d\sigma^2_{x_i}}{dt}=2\left(\int x_iU_{ij}\psi_idx_i-m_{x_i}\int U_{ij}\psi_idx_i\right)+\mu_{ij}=2\,\mathrm{cov}_{x_i,\dot x_i}+\mu_{ij},$$

$$\frac{d\sigma^2_{x_j}}{dt}=2\left(\int x_jV_{ij}\psi_jdx_j-m_{x_j}\int V_{ij}\psi_jdx_j\right)+\kappa_{ij}=2\,\mathrm{cov}_{x_j,\dot x_j}+\kappa_{ij}. \tag{14}$$

Expressions (14) reveal the statistical contents of parameters $\mu_{ij}$ and $\kappa_{ij}$, which are equal to the difference between the rate of change of variance and twice the covariance between indicators and their first differences. After substitution in (14) expressions (10) evolution equation dispersions can also be represented as:

$$\frac{d\sigma_{x_i}^2}{dt}=2\lambda_{ij}\left(\sigma_{x_i}^2\left(1-m_{x_j}\right)+m_{x_i}\,cov_{x_i,x_j}-cov_{x_i^2,x_j}\right)+\mu_{ij},$$
$$\frac{d\sigma_{x_j}^2}{dt}=2\chi_{ij}\left(\sigma_{x_j}^2\left(1-m_{x_i}\right)+m_{x_j}\,cov_{x_i,x_j}-cov_{x_i,x_j^2}\right)+\kappa_{ij}. \tag{15}$$

Evolution of covariance between economic parameters is determined from the equation:

$$\frac{dR_{ij}}{dt}=\int\left(x_i-m_{x_i}\right)\left(x_j-m_{x_j}\right)\frac{\partial\varphi_{ij}\left(x_ix_j;t\right)}{\partial t}dx_idx_j=$$
$$=\int\left(x_i-m_{x_i}\right)\left(x_j-m_{x_j}\right)\left(\frac{\mu_{ij}}{2}\frac{\partial^2\varphi_{ij}}{\partial x_i^2}+\eta_{ij}\frac{\partial^2\varphi_{ij}}{\partial x_i\partial x_j}+\frac{\kappa_{ij}}{2}\frac{\partial^2\varphi_{ij}}{\partial x_j^2}-\frac{\partial}{\partial x_i}\left(u_{ij}\varphi_{ij}\right)-\frac{\partial}{\partial x_j}\left(v_{ij}\varphi_{ij}\right)\right)dx_idx_j.$$

After integration we obtain the following equation of evolution $R_{ij}(t)$:

$$\frac{dR_{ij}}{dt}=\eta_{ij}+\int x_ju_{ij}\varphi_{ij}dx_idx_j+\int x_iv_{ij}\varphi_{ij}dx_idx_j-m_{x_j}\int u_{ij}\varphi_{ij}dx_idx_j-$$
$$-m_{x_i}\int v_{ij}\varphi_{ij}dx_idx_j=\eta_{ij}+\mathrm{cov}_{x_i,\dot{x}_j}+\mathrm{cov}_{x_j,\dot{x}_i}. \tag{16}$$

Equation (16) explains the statistical meaning of the parameter $\eta_{ij}$, what is the difference between the rate of change of the covariance parameters and the covariance sum coordinate between one indicator and another indicator rate. In expression (16) taking into account the covariance model equations (7) can be presented as:

$$cov_{x_i,\dot{x}_j}=\lambda_{ij}\left(cov_{x_i,x_j}\left(1+m_{x_j}\right)-cov_{x_i,x_j^2}-m_{x_i}\sigma_{x_j}^2\right),$$
$$cov_{x_i,\dot{x}_j}=\chi_{ij}\left(cov_{x_i,x_j}\left(1+m_{x_i}\right)-cov_{x_j,x_i^2}-m_{x_j}\sigma_{x_i}^2\right). \tag{17}$$

So if at time to perform statistical evaluation of mathematical expectations and variances for covariance sample data, then according to (13), (15) and (16), (17), the parameters of the two-dimensional Fokker - Planck will be fully defined.

*3. Predictive model of the stock exchange*. In the study of multivariate processes usually only have limited knowledge about the first two moments: average values of the vector and covariance matrix. Therefore, to predict multivariate ETS should be used one-dimensional scheme.

The prediction parameters of trading will be based on kinetic equations of evolution:

$$\dot{x}_i=\lambda_{ij}x_i\left(1-e_{ij}\right)-\frac{\mu_{ij}}{2\psi_i}\frac{\partial\psi_i}{\partial x_i},\quad \dot{x}_j=\chi_{ij}x_j\left(1-f_{ij}\right)-\frac{\kappa_{ij}}{2\psi_j}\frac{\partial\psi_j}{\partial x_j}, \tag{18}$$

where $\psi_i=\psi_i(x_i,t)$, $\psi_j=\psi_j\left(x_j,t\right)$ – one-dimensional (partial) PDF, satisfying one-dimensional Fokker - Planck equation (9). Fokker-Planck based model equations (7) are transformed to the form:

$$\frac{\partial\psi_i}{\partial t}+\lambda_{ij}\left(\psi_i+x_i\frac{\partial\psi_i}{\partial x_i}\right)-\frac{\mu_{ij}}{2}\frac{\partial^2\psi_i}{\partial x_i^2}=\lambda_{ij}\left(E_{ij}+x_i\frac{\partial E_{ij}}{\partial x_i}\right),$$
$$\frac{\partial\psi_j}{\partial t}+\chi_{ij}\left(\psi_j+x_j\frac{\partial\psi_j}{\partial x_j}\right)-\frac{\kappa_{ij}}{2}\frac{\partial^2\psi_j}{\partial x_j^2}=\chi_{ij}\left(F_{ij}+x_j\frac{\partial F_{ij}}{\partial x_j}\right), \tag{19}$$

where $E_{ij}=e_{ij}(x_i,t)\cdot\psi_i(x_i,t)$, $F_{ij}=f_{ij}\left(x_j,t\right)\cdot\psi_j\left(x_j,t\right)$ – unknown stream function, which according to (11) are determined through a two-dimensional PDF.

To determine the unknown functions $E_{ij}$, $F_{ij}$ we used the idea of building the connected evolution equations for higher order moments for closure system that proposed in [11]. Depending on the order of points in PDF is a more or less complex dynamic system that models the ETS. For the trade models considered here the following equations, consistent with the two-dimensional Fokker - Planck equation (6) will be:

$$\frac{\partial E_{ij}}{\partial t}+\lambda_{ij}x_i\frac{\partial E_{ij}}{\partial x_i}+\left(\lambda_{ij}-\chi_{ij}\left(1-x_i\right)\right)E_{ij}-\frac{\mu_{ij}}{2}\frac{\partial^2 E_{ij}}{\partial x_i^{\,2}}=\lambda_{ij}\left(G_{ij}+x_i\frac{\partial G_{ij}}{\partial x_i}\right)-\eta_{ij}\frac{\partial \psi_i}{\partial x_i}, \tag{20}$$

$$\frac{\partial F_{ij}}{\partial t}+\chi_{ij}x_j\frac{\partial F_{ij}}{\partial x_j}+\left(\chi_{ij}-\lambda_{ij}\left(1-x_j\right)\right)F_{ij}-\frac{\kappa_{ij}}{2}\frac{\partial^2 F_{ij}}{\partial x_j^{\,2}}=\chi_{ij}\left(H_{ij}+x_j\frac{\partial H_{ij}}{\partial x_j}\right)-\eta_{ij}\frac{\partial \psi_j}{\partial x_j},$$

in which now the unknown are the function $G_{ij}=g_{ij}(x_i,t)\psi_i(x_i,t)$ and $H_{ij}=h_{ij}\left(x_j,t\right)\psi_j\left(x_j,t\right)$, and second order moments are determined by the formulas:

$$g_{ij}\left(x_i,t\right)=\frac{1}{\psi_i\left(x_i,t\right)}\int x_j^2\varphi_{ij}\left(x_i,x_j,t\right)dx_j,\quad h_{ij}=\frac{1}{\psi_j\left(x_j,t\right)}\int x_i^2\varphi_{ij}\left(x_i,x_j,t\right)dx_i. \tag{21}$$

If at this moment close the system, and $g_{ij}$ , $h_{ij}$ specify independently, the evolution of the system that generates multivariate ETS, will be fully defined. The content of the expression (21), as well as the expression (11) is the conditional mathematical expectation - regression function that can be recovered for the period $[t-T,t]$, prior to the forecast, of regression analysis. Here $T$ – length of series or interval of observations. Discrete scheme for solving the problem (18), (19), (20) is based on an explicit difference scheme for the evolution in time and standard templates left difference derivatives

*x:*

$$\tilde{x}_i(t+1)=x_i(t)\left(1+\left(1-e_{ij}(x_i,t-1)\right)\lambda_{ij}(t-1)\right)-\frac{\mu_{ij}(t-1)}{2\psi_i(x_i,t)}\left(\psi_i(x_i+1,t)-\psi_i(x_i,t)\right),$$

$$\tilde{x}_j(t+1)=x_j(t)\left(1+\left(1-f_{ij}(x_j,t-1)\right)\chi_{ij}(t-1)\right)-\frac{\kappa_{ij}(t-1)}{2\psi_j(x_j,t)}\left(\psi_j(x_j+1,t)-\psi_j(x_j,t)\right);$$

$\psi$ :

$$\begin{aligned}\tilde{\psi}_i\left(x_i,t\right)=&\psi_i\left(x_i,t-1\right)+\frac{\mu_{ij}(t-1)}{2}\left(\psi_i\left(x_i+2,t-1\right)-2\psi_i\left(x_i+1,t-1\right)+\psi_i\left(x_i,t-1\right)\right)-\\&-\lambda_{ij}(t-1)\left(\psi_i\left(x_i,t-1\right)+x_i(t-1)\left(\psi_i\left(x_i+1,t-1\right)-\psi_i\left(x_i,t-1\right)\right)\right)+\\&+\lambda_{ij}(t-1)\left(E_{ij}\left(x_i,t-1\right)+x_i(t-1)\left(E_{ij}\left(x_i+1,t-1\right)-E_{ij}\left(x_i,t-1\right)\right)\right),\end{aligned}$$

$$\begin{aligned}\tilde{\psi}_j\left(x_j,t\right)=&\psi_j\left(x_j,t-1\right)+\frac{\kappa_{ij}(t-1)}{2}\left(\psi_j\left(x_j+2,t-1\right)-2\psi_j\left(x_j+1,t-1\right)+\psi_j\left(x_i,t-1\right)\right)-\\&-\chi_{ij}(t-1)\left(\psi_j\left(x_i,t-1\right)+x_j(t-1)\left(\psi_j\left(x_i+1,t-1\right)-\psi_j\left(x_j,t-1\right)\right)\right)+\\&+\chi_{ij}(t-1)\left(F_{ij}\left(x_j,t-1\right)+x_j(t-1)\left(F_{ij}\left(x_j+1,t-1\right)-E_{ij}\left(x_j,t-1\right)\right)\right);\end{aligned} \tag{22}$$

$E$ :

$$\begin{aligned}\tilde{E}_{ij}(x_i,t-1)=&E_{ij}(x_i,t-2)+\frac{\mu_{ij}(t-2)}{2}\left(E_{ij}(x_i+2,t-2)-2E_{ij}(x_i+1,t-2)+E_{ij}(x_i,t-2)\right)-\\&-\lambda_{ij}(t-2)x_i(t-2)\left(E_{ij}(x_i+1,t-2)-E_{ij}(x_i,t-2)\right)-\left(\lambda_{ij}(t-2)-\left(1-x_i(t-2)\right)\chi_{ij}(t-2)\right)E_{ij}(x_i,t-2)+\\&+\lambda_{ij}(t-2)\left(G_{ij}(x_i,t-2)+x_i(t-2)\left(G_{ij}(x_i+1,t-2)-G_{ij}(x_i,t-2)\right)\right)-\eta_{ij}(t-2)\left(\psi_i(x_i+1,t-2)-\psi_i(x_i,t-2)\right);\end{aligned}$$

$F$ :

$$\tilde{F}_{ij}(x_j,t-1)=F_{ij}(x_j,t-2)+\frac{\kappa_{ij}(t-2)}{2}\left(F_{ij}(x_j+2,t-2)-2E_{ij}(x_j+1,t-2)+F_{ij}(x_j,t-2)\right)-$$
$$-\chi_{ij}(t-2)x_j(t-2)\left(F_{ij}(x_j+1,t-2)-F_{ij}(x_j,t-2)\right)-\left(\chi_{ij}(t-2)-(1-x_j(t-2))\lambda_{ij}(t-2)\right)F_{ij}(x_j,t-2)+$$
$$+\chi_{ij}(t-2)\left(H_{ij}(x_j,t-2)+x_j(t-2)\left(H_{ij}(x_j+1,t-2)-H_{ij}(x_j,t-2)\right)\right)-\eta_{ij}(t-2)\left(\psi_j(x_j+1,t-2)-\psi_j(x_j,t-2)\right),$$

where $\tilde{x}_i$, $\tilde{x}_j$, $\tilde{\psi}_i$, $\tilde{\psi}_j$, $\tilde{E}_{ij}$, $\tilde{F}_{ij}$ - predicted value.

According to the system (22) forecasting parameters of trading and PDF $\psi_i$ and $\psi_j$ for one step forward, starting from the last equations for which right sides are known already. System (22) can be also used for forecasting several steps forward. At the same time every solution found in the next step joins the sliding window sampling, which is based on the Fokker - Planck equation. In this case, the initial conditions of the problem are put at the start of forecasting, and the boundary conditions are taken zero.

4. *The modeling of time series.* Verification of predictive model (22) was performed by dynamic series, formed according to the daily trading on the PFTS security UNAF in period 26.03.2009 to 30.09.2013 with: $x_1$ – average quotation price, UAH. $x_2$ – physical volume of sales, pc $x_3$ – spread, %. The total number of observations $n$ = 1122. Figure 1 shows a series of normalized indices trading

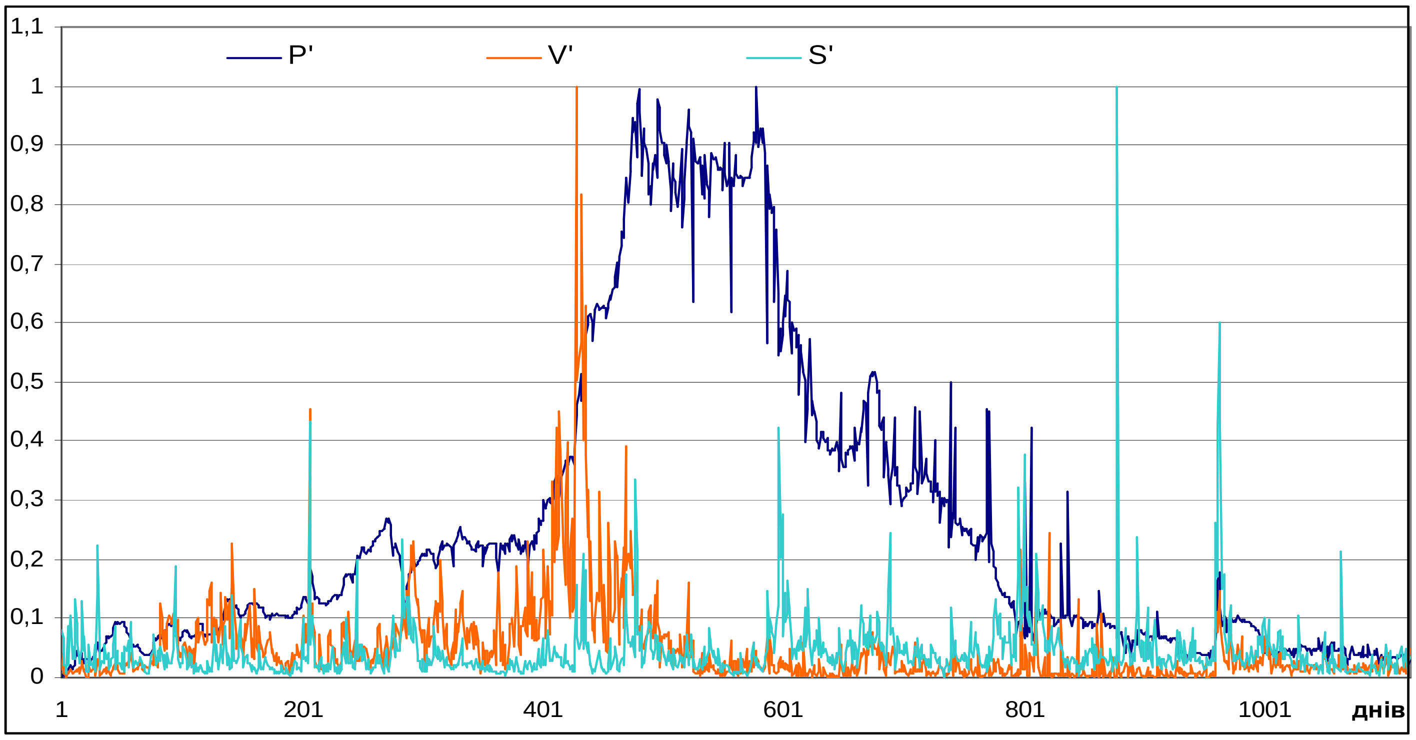


*Figure 1. Dynamics of basic indices trading securities UNAF*

The minimum values of the output units amounted to 83.41 UAH, 109 units., 0%, and the maximum - 908.85 UAH, 335 398 units, 96.88% respectively. Rms relative deviations of trades amounted to 216.69 UAH, 25 911 units, 5.45%, and the mean square deviation of normalized values - 0.26, 0.08, 0.06, respectively. Then for a sample length $n$ = 200 forecast PDF in $\tau$ = 26 steps forward, according to estimates $\varepsilon \geq 2\tau / n$ [11], guaranteed with an accuracy smaller than most of the found values of mean-square deviation, that is $\varepsilon = 0.26$.

Accuracy of PDF and the information content of PDF are also dependent on the number of intervals partitioning the histogram. It is believed that the optimum number of intervals to be $k \approx 1 + 3{,}32 \lg n$ (Sturges formula). On the other hand, the length of the sampling interval dependent stability PDF explicit difference scheme, used in the model (22). Stability of solutions, which obtained difference approximation equations of the form (9) is provided under condition $\Delta t < (\Delta x)^2 / b$, where $b$ – diffusion coefficient. In our case, when $x_i \in [0,1]$, a $\Delta t = 1$, obtain the following estimate the upper limit of the number of intervals PDF: $k < 1/\sqrt{b}$, where $b = \max_t \{\mu_{ij} \kappa_{ij}\}$. The resulting values $b$=0.092 for number of intervals PDF was 10 that does not contradict the formula Sturges.

Simulations carried out by a number of numerical integration of equations (22), 30 steps ahead. In this case, the best prediction accuracy achieved by the criterion of MAPE (Mean absolute percentage error) accounted for 3.71% price quotes, trading volume - 2.18 % and spread - 1.37 %. Based on the methodology of modeling multivariate dynamic systems obtained new scientific results - first constructed multivariate stochastic model parameters ETS trading on the stock exchange, thus improving the accuracy and extend the time horizon for the medium-term. Assumed that in the future, this model will be used in the development of an information model investment decisions in the stock market.

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